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A landscape of non-supersymmetric AdS vacua on coset manifolds

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ABSTRACT: We construct new families of non-supersymmetric sourceless type IIA AdS_4 vacua on those coset manifolds that also admit supersymmetric solutions. We investigate the spectrum of left-invariant modes and find that most, but not all, of the vacua are stable under these fluctuations. Generically, there are also no massless moduli.

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Contents

1. Introduction

The reasons for studying AdS_4 vacua of type IIA supergravity are twofold: first they are examples of flux compactifications away from the Calabi-Yau regime, where all the moduli can be stabilized at the classical level. Secondly, they can serve as a gravity dual in the AdS_4/CFT_3-correspondence, which became the focus of attention due to recent progress in the understanding of the CFT-side as a Chern-Simons-matter theory describing the world-volume of coinciding M2-branes [1].

It is much easier to find supersymmetric solutions of supergravity as the supersymmetry conditions are simpler than the full equations of motion, while at the same time there are general theorems stating that the former – supplemented with the Bianchi identities of the form fields – imply the latter [2, 3, 4, 5]. Although special type IIA solutions that came from the reduction of supersymmetric M-theory vacua were already known (see e.g. [6, 7, 8]), it was only in [3] that the supersymmetry conditions for type IIA vacua with SU(3)-structure were first worked out in general. It was discovered that there are natural solutions to these equations on the four coset manifolds G/H that have a nearly-Kähler limit [9, 10, 11, 12, 13, 14] (solutions on other manifolds can be found in e.g. [3, 15, 16]).[1] To be precise these are the manifolds $SU(2) \times SU(2)$, $\frac{G_2}{SU(3)}$, $\frac{Sp(2)}{S(U(2) \times U(1))}$ and $\frac{SU(3)}{U(1) \times U(1)}$.[2] These solutions are particularly simple in the sense that both the SU(3)-structure, which determines the metric, as well as all the form fluxes can be expanded in terms of forms which are left-invariant under the action of the group G. The supersymmetry equations

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[1]For an early appearance of these coset manifolds in the string literature see e.g. [17].

[2]See [18] for a review and a proof that these are the only homogeneous manifolds admitting a nearly-Kähler geometry.

of [3] then reduce to purely algebraic equations and can be explicitly solved. Nevertheless, these solutions still have non-trivial geometric fluxes as opposed to the Calabi-Yau or torus orientifolds of [15, 16]. Similarly to those papers it is possible to classically stabilize all left-invariant moduli [14]. Inspired by the $\text{AdS}_4/\text{CFT}_3$ correspondence more complicated type IIA solutions have in the meantime been proposed. The solutions have a more generic form for the supersymmetry generators, called $\text{SU}(3) \times \text{SU}(3)$-structure [19], and are not left-invariant anymore [20, 21, 22, 23] (see also [24]). Supersymmetric AdS_4 vacua in type IIB with $\text{SU}(2)$-structure have also been studied in [25, 26, 27, 28] and in particular it has been shown in [28] that also in this setup classical moduli stabilization is possible.

At some point, however, supersymmetry has to be broken and we have to leave the safe haven of the supersymmetry conditions. In this paper we construct new *non-supersymmetric* AdS_4 vacua without source terms. This means that the more complicated equations of motion of supergravity should be tackled directly[3]. In order to simplify the equations we use a specific ansatz: we start from a supersymmetric AdS_4 solution and scan for non-supersymmetric solutions with the *same* geometry (and thus $\text{SU}(3)$-structure), but with *different* NSNS- and RR-fluxes. Moreover, we expand these form fields in terms of the $\text{SU}(3)$-structure and its torsion classes. This may seem restrictive at first, but it works for 11D supergravity, where solutions like this have been found and are known as Englert-type solutions [31, 32, 33] (see [34] for a review). To be specific, for each supersymmetric M-theory solution of Freund-Rubin type (which means the M-theory four-form flux has only legs along the external AdS_4 space, i.e. $F_4 = f\text{vol}_4$ where f is called the Freund-Rubin parameter) it is possible to construct a non-supersymmetric solution with the same internal geometry but with a different four-form flux. The modified four-form of the Englert solution has then a non-zero internal part: $\hat{F}_4 \propto \eta^\dagger \gamma_{m_1 m_2 m_3 m_4} \eta \, \mathrm{d}x^{m_1 m_2 m_3 m_4}$, where η is the 7D supersymmetry generator, and a different Freund-Rubin parameter $f_E = -(2/3)f$. Also the Ricci scalar of the AdS_4 space, and thus the effective 4D cosmological constant, differs: $R_{\text{4D},E} = (5/6)R_{\text{4D}}$. In type IIA with non-zero Romans mass (so that there is no lift to M-theory) non-supersymmetric solutions of this form have been found as well: for the nearly-Kähler geometry in [35, 29, 36] and for the Kähler-Einstein geometry in [35, 20, 37]. In this paper we show that this type of solutions is not restricted to these limits and systematically scan for them. Applying our ansatz to the coset manifolds with nearly-Kähler limit, mentioned above, we find that the most interesting manifolds are $\frac{\text{Sp}(2)}{\text{S}(\text{U}(2)\times\text{U}(1))}$ and $\frac{\text{SU}(3)}{\text{U}(1)\times\text{U}(1)}$, on which we find several families of non-supersymmetric AdS_4 solutions. We also find some non-supersymmetric solutions in regimes of the geometry that do not allow for a supersymmetric solution.

These non-supersymmetric solutions are not necessarily stable. For instance, it is known that if there is more than one Killing spinor on the internal manifold (which holds in particular for S^7, the M-theory lift of $\mathbb{CP}^3 = \frac{\text{Sp}(2)}{\text{S}(\text{U}(2)\times\text{U}(1))}$), the Englert-type solution is unstable [38]. We investigate stability of our solutions against left-invariant fluctuations. This means we calculate the spectrum of left-invariant modes, and check for each mode

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[3]Another route would be to find some alternative first-order equations, which extend the supersymmetry conditions in that they still automatically imply the full equations of motion in certain non-supersymmetric cases, see e.g. [29, 30].

whether the mass-squared is above the Breitenlohner-Freedman bound [39, 40]. This is not a complete stability analysis in that there could still be non-left-invariant modes that are unstable. We do believe it provides a good first indication. In particular, we find for the type IIA reduction of the Englert solution on S^7 that the unstable mode of [38] is among our left-invariant fluctuations and we find the exact same mass-squared.

These non-supersymmetric AdS$_4$ vacua are interesting, because, provided they are stable, they should have a CFT-dual. For instance in [20] the CFT-dual for a non-supersymmetric Kähler-Einstein solution on $\mathbb{CP}^3$ was proposed. Furthermore, for phenomenologically more realistic vacua, supersymmetry-breaking is essential. Really, one would like to construct classical solutions with a dS$_4$-factor, which are necessarily non-supersymmetric. Because of a series of no-go theorems – from very general to more specific: [41, 42, 43, 44, 45] – this is a very non-trivial task. For papers nevertheless addressing this problem see [46, 47, 45, 48, 49, 28]. In this context the landscape of the non-supersymmetric AdS$_4$ vacua of this paper can be considered as a playground to gain experience before trying to construct dS$_4$-vacua. In fact, in [48] an ansatz very similar to the one used in this paper was proposed in order to construct dS$_4$-vacua. Applied to the coset manifolds above, it did however not yield any solutions, in agreement with the no-go theorem of [45].

In section 2 we explain our ansatz in full detail, while in section 3 we present the explicit solutions we found on the coset manifolds. In section 4 we analyse the stability against left-invariant fluctuations before ending with some short conclusions. We provide an appendix with some useful formulae involving SU(3)-structures and an appendix on our supergravity conventions.

The non-supersymmetric solutions of this paper appeared before in the second author's PhD thesis [50].

2. Ansatz

In this section we explain the ansatz for our non-supersymmetric solutions. The reader interested in the details might want to check out our SU(3)-structure conventions in appendix A, while towards the end of the section we need the type II supergravity equations of motion outlined in appendix B.

We start with a supersymmetric SU(3)-structure solution of type IIA supergravity. The SU(3)-structure is defined by a real two-form J and a complex decomposable three-form Ω satisfying (A.1). Moreover, J and Ω together determine the metric as in (A.2). In order for the solution to preserve at least one supersymmetry ($N = 1$) [3] one finds that the warp factor A and the dilaton Φ should be constant, the torsion classes $\mathcal{W}_1, \mathcal{W}_2$ purely imaginary and all other torsion classes zero (for the definition of the torsion classes see (A.3)). This implies

$$\mathrm{d}J = \frac{3}{2}W_1 \mathrm{Re}\,\Omega\,, \tag{2.1a}$$

$$\mathrm{d}\mathrm{Re}\,\Omega = 0\,, \tag{2.1b}$$

$$\mathrm{d}\mathrm{Im}\,\Omega = W_1 J \wedge J + W_2 \wedge J\,, \tag{2.1c}$$

where we defined $W_1 \equiv -i\mathcal{W}_1$ and $W_2 \equiv -i\mathcal{W}_2$. The fluxes can then be expressed in terms of Ω, J and the torsion classes and are given by

$$e^\Phi \hat{F}_0 = f_1 \,, \tag{2.2a}$$

$$e^\Phi \hat{F}_2 = f_2 \, J + f_3 \, \hat{W}_2 \,, \tag{2.2b}$$

$$e^\Phi \hat{F}_4 = f_4 \, J \wedge J + f_5 \, \hat{W}_2 \wedge J \,, \tag{2.2c}$$

$$e^\Phi \hat{F}_6 = f_6 \, \mathrm{vol}_6 \,, \tag{2.2d}$$

$$H = f_7 \, \mathrm{Re}\,\Omega \,, \tag{2.2e}$$

where for the supersymmetric solution

$$f_1 = e^\Phi m \,, \quad f_2 = -\frac{W_1}{4} \,, \quad f_3 = -w_2 \,, \quad f_4 = \frac{3\, e^\Phi m}{10} \,,$$

$$f_5 = 0 \,, \quad f_6 = \frac{9\, W_1}{4} \,, \quad f_7 = \frac{2\, e^\Phi m}{5} \,. \tag{2.3}$$

Using the duality relation $f = \tilde{F}_0 = -\star_6 \hat{F}_6 = -e^{-\Phi} f_6$ (see (B.6)) we find that f_6 is proportional to the Freund-Rubin parameter f, while f_1 is proportional to the Romans mass m. Furthermore, we introduced here a normalized version of W_2, enabling us later on to use (2.2) as an ansatz for the fluxes also in the limit $W_2 \to 0$:

$$\hat{W}_2 = \frac{W_2}{w_2} \,, \quad \text{with} \quad w_2 = \pm\sqrt{(W_2)^2} \,, \tag{2.4}$$

where one can choose a convenient sign in the last expression.

The Bianchi identity for $\hat{F}_2$ imposes $\mathrm{d}W_2 \propto \mathrm{Re}\,\Omega$. Working out the proportionality constant [3] we find

$$\mathrm{d}W_2 = -\frac{1}{4}(W_2)^2 \mathrm{Re}\,\Omega \,. \tag{2.5}$$

Furthermore, using the values for the fluxes (2.3) it fixes the Romans mass:

$$e^{2\Phi} m^2 = \frac{5}{16}\left(3(W_1)^2 - 2(W_2)^2\right) \,. \tag{2.6}$$

We now want to construct non-supersymmetric AdS solutions on the manifolds mentioned in the introduction with the *same* geometry as in the supersymmetric solution, and thus the same SU(3)-structure (J, Ω), but with *different* fluxes. We make the ansatz that the fluxes can still be expanded in terms of J, Ω and the torsion class $\hat{W}_2$ as in (2.2), but with different values for the coefficients f_i. To this end we plug the ansatz for the geometry (J, Ω) — eqs. (2.1) — and the ansatz for the fluxes — eqs. (2.2) — into the equations of motion (B.7) and solve for the f_i. We will make one more assumption, namely that

$$\hat{W}_2 \wedge \hat{W}_2 = c\, J \wedge J + p\, \hat{W}_2 \wedge J \,, \tag{2.7}$$

with c, p some parameters. This is an extra constraint only for the $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ coset and we will discuss its relaxation later.[4] Wedging with J we find then immediately $c = -1/6$.

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[4]With the ansatz (2.2) the constraint is forced upon us. Indeed, suppose that instead $\hat{W}_2 \wedge \hat{W}_2 = -1/6\, J \wedge J + p\hat{W}_2 \wedge J + P \wedge J$, where P is a non-zero simple (1,1)-form independent of $\hat{W}_2$. We find then from the equation of motion for H and the internal part of the Einstein equation respectively $f_5 f_3 = 0$ and $(f_3)^2 - (f_5)^2 - (w_2)^2 = 0$. So the only possibility is then $f_5 = 0$ and $f_3 = \pm w_2$, which leads in the end to the supersymmetric solution. They way out is to also include P as an expansion form in (2.2).

Furthermore we need expressions for the Ricci scalar and tensor, which for a manifold with SU(3)-structure can be expressed in terms of the torsion classes [51]. Taking into account that only $W_{1,2}$ are non-zero we find:

$$R_{6D} = \frac{15\,(W_1)^2}{2} - \frac{(W_2)^2}{2}\,, \tag{2.8a}$$

$$R_{mn} = \frac{1}{6}g_{mn}R_{6D} + \frac{W_1}{4}\,W_{2(m}\cdot J_{n)} + \frac{1}{2}\,[W_{2\,m}\cdot W_{2\,n}]_0 + \frac{1}{2}\mathrm{Re}\left[\mathrm{d}W_2|_{(2,1)\,m}\cdot\bar{\Omega}_n\right]\,, \tag{2.8b}$$

where $(P)^2$ and $P_m\cdot P_n$ for a form P are defined in (B.2) and $|_0$ indicates taking the traceless part. From eq. (2.5) follows that for our purposes $\mathrm{d}W_2|_{2,1} = 0$ so that the last term in (2.8b) vanishes. Moreover, using (2.7) $[W_{2\,m}\cdot W_{2\,n}]_0$ can be expressed in terms of $W_{2(m}\cdot J_{n)}$.

Plugging the ansatz for the fluxes (2.2) into the equations of motion (B.7) and using eqs. (2.1), (2.5), (A.5), (2.7), (B.5), (B.6) and (2.8b) we find:

$$\text{Bianchi } F_2:\;\; 0 = \frac{3}{2}\,W_1 f_2 - \frac{1}{4}w_2 f_3 + f_1 f_7\,,$$

$$\text{eom } F_4:\;\; 0 = 3\,W_1 f_4 + \frac{1}{4}w_2 f_5 - f_6 f_7\,,$$

$$\text{eom } H:\;\; 0 = 6\,W_1 f_7 - 3 f_1 f_2 - 12 f_4 f_2 - 6 f_4 f_6 - f_3 f_5\,,$$

$$0 = w_2 f_7 + f_1 f_3 + f_2 f_5 - 2 f_3 f_4 - f_5 f_6 + p f_3 f_5\,, \tag{2.9}$$

$$\text{dilaton eom}:\;\; 0 = R_{4D} + R_{6D} - 2 f_7^2\,,$$

$$\text{Einstein ext.}:\;\; 0 = R_{4D} + (f_1)^2 + 3(f_2)^2 + 12(f_4)^2 + (f_6)^2 + (f_3)^2 + (f_5)^2\,,$$

$$\text{Einstein int.}:\;\; 0 = R_{6D} - 6(f_7)^2 + \frac{1}{2}\left[3(f_1)^2 + 3(f_2)^2 - 12(f_4)^2 - 3(f_6)^2 + (f_3)^2 - (f_5)^2\right]\,,$$

$$0 = 4(f_2 f_3 + 2 f_4 f_5) - w_2 W_1 - p\left[(f_3)^2 - (f_5)^2 - (w_2)^2\right]\,.$$

In the equation of motion for H we get separate conditions from the coefficients of $J\wedge J$ and $\hat{W}_2\wedge J$ respectively. In the internal Einstein equation we find likewise a separate condition from the trace and the coefficient of $W_{2\,(m}\cdot J_{n)}$. In the next section we find explicit solutions to these equations for the coset manifolds with nearly-Kähler limit, the stability of which we investigate in section 4.

Flipping signs

The Einstein and dilaton equation are quadratic in the form fluxes and thus insensitive to flipping the signs of these fluxes. Taking into account also the flux equations of motion and Bianchi identities, we find that for each solution to the supergravity equations, we automatically obtain new ones by making the following sign flips:

$$H \to -H\,,\quad \hat{F}_0 \to -\hat{F}_0\,,\quad \hat{F}_2 \to \hat{F}_2\,,\quad \hat{F}_4 \to -\hat{F}_4\,,\quad \hat{F}_6 \to \hat{F}_6\,,$$

$$H \to -H\,,\quad \hat{F}_0 \to \hat{F}_0\,,\quad \hat{F}_2 \to -\hat{F}_2\,,\quad \hat{F}_4 \to \hat{F}_4\,,\quad \hat{F}_6 \to -\hat{F}_6\,, \tag{2.10}$$

$$H \to H\,,\quad \hat{F}_0 \to -\hat{F}_0\,,\quad \hat{F}_2 \to -\hat{F}_2\,,\quad \hat{F}_4 \to -\hat{F}_4\,,\quad \hat{F}_6 \to -\hat{F}_6\,.$$

In particular, these sign flips will transform a supersymmetric solution into another supersymmetric solution (as can be verified using the conditions (2.1),(2.3) allowing for suitable

sign flips of J, $\mathrm{Re}\,\Omega$ and $\mathrm{Im}\,\Omega$ compatible with the metric). If some fluxes are zero, more sign flips are possible. For instance for $\hat{F}_0 = \hat{F}_4 = 0$ we find the following extra sign-flip, known as *skew-whiffing* in the M-theory compactification literature [52] (see also the review [34])

$$H \to \pm H\,, \quad \hat{F}_2 \to \hat{F}_2\,, \quad \hat{F}_6 \to -\hat{F}_6\,, \tag{2.11}$$

which transforms a supersymmetric solution into a *non*-supersymmetric one. When discussing different solutions, we will from now on implicitly consider each solution together with its signed-flipped counterparts.

3. Solutions

Let us now solve the equations obtained in the previous section for the coset manifolds that admit sourceless supersymmetric solutions, namely $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\mathrm{SU}(2){\times}\mathrm{SU}(2)$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2){\times}\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1){\times}\mathrm{U}(1)}$. For the supersymmetric solutions on these manifolds we will use the conventions and presentation of [13, 14]. For more details, including in particular our choice of structure constants for the relevant algebras, we refer to these papers.

On a coset manifold G/H one can define a coframe e^m through the decomposition of the Lie-valued one-form $L^{-1}\mathrm{d}L = e^m \mathcal{K}_m + \omega^a \mathcal{H}_a$ in terms of the algebras of G and H. Here L is a coset representative, the $\mathcal{H}_a$ span the algebra of H and the $\mathcal{K}_m$ span the complement of this algebra within the algebra of G. The exterior derivative on the e^m is then given in terms of the structure constants through the Maurer-Cartan relation. Furthermore, the forms that are *left-invariant* under the action of G are precisely those forms that are constant in the basis spanned by e^m and for which the exterior derivative is also constant in this basis. For these forms the exterior derivative can then be expressed solely in terms of the structure constants only involving the $\mathcal{K}_m$. We refer to [53, 54] for a review on coset technology or to the above papers for a quick explanation.

$\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ and $\mathrm{SU}(2){\times}\mathrm{SU}(2)$

We start from the supersymmetric nearly-Kähler solution on $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$. The SU(3)-structure is given by

$$
\begin{aligned}
J &= a(e^{12} - e^{34} + e^{56})\,, \\
\Omega &= a^{3/2}\left[(e^{245} - e^{236} - e^{146} - e^{135}) + i(e^{246} + e^{235} + e^{145} - e^{136})\right]\,,
\end{aligned}
\tag{3.1}
$$

where a is the overall scale.

Since this SU(3)-structure corresponds to a nearly-Kähler geometry the torsion class W_2 is zero. Furthermore we find

$$W_1 = -\frac{2}{\sqrt{3}}a^{-1/2}\,, \quad w_2 = p = 0\,. \tag{3.2}$$

Plugging this into the equations (2.9) we find exactly three solutions for $(f_1, \ldots, f_7)$ (up to the sign flips (2.10)):

$$
\begin{aligned}
& a^{-1/2}\Big(\frac{\sqrt{5}}{2}, \frac{1}{2\sqrt{3}}, 0, \frac{3}{4\sqrt{5}}, 0, -\frac{9}{2\sqrt{3}}, \frac{1}{\sqrt{5}}\Big)\,, \\
& a^{-1/2}\Big(\sqrt{\frac{5}{3}}, 0, 0, 0, 0, \frac{5}{\sqrt{3}}, 0\Big)\,, \\
& a^{-1/2}\Big(1, \frac{1}{\sqrt{3}}, 0, -\frac{1}{2}, 0, \sqrt{3}, 1\Big)\,.
\end{aligned}
\tag{3.3}
$$

The first is the supersymmetric solution, while the last two are non-supersymmetric solutions, which were already found in [35, 29, 36]. Truncating to the 4D effective theory it was shown in [30] that a generalization of this family of solutions is quite universal as it appears in a large class of $N = 2$ gauged supergravities.

On the SU(2)×SU(2) manifold, requiring the same geometry as the supersymmetric solution and not allowing for source terms will restrict us to the nearly-Kähler point. The analysis is then basically the same as for $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ above.

$\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$

The family of supersymmetric solutions on this manifold has, next to the overall scale, an extra parameter determining the shape of the solutions. It is then possible to turn on the torsion class W_2 and venture away from the nearly-Kähler geometry. This makes this class much richer and enables us this time to find new non-supersymmetric solutions. The SU(3)-structure is given by [12, 13, 14]

$$
\begin{aligned}
J &= a(e^{12} + e^{34} - \sigma e^{56})\,, \\
\Omega &= a^{3/2}\sigma^{1/2}\left[(e^{245} - e^{236} - e^{146} - e^{135}) + i(e^{246} + e^{235} + e^{145} - e^{136})\right]\,,
\end{aligned}
\tag{3.4}
$$

where a is the overall scale and σ is the shape parameter. We find for the torsion classes and the parameter p:

$$
\begin{aligned}
W_1 &= (a\sigma)^{-1/2}\frac{2+\sigma}{3}\,, \\
(W_2)^2 &= (a\sigma)^{-1}\frac{8(1-\sigma)^2}{3} \Rightarrow w_2 = (a\sigma)^{-1/2}\frac{2\sqrt{2}(1-\sigma)}{\sqrt{3}}\,, \\
\hat{W}_2 &= -\frac{1}{\sqrt{3}}\left(e^{12} + e^{34} + 2\sigma e^{56}\right)\,, \\
p &= -\sqrt{2/3}\,.
\end{aligned}
\tag{3.5}
$$

We easily read off that $\sigma = 1$ corresponds to the nearly-Kähler geometry. Note that even though $W_2 \to 0$ for $\sigma \to 1$, $\hat{W}_2$ is well-defined and non-zero in this limit so that we can still use it as an expansion form for the fluxes. The points $\sigma = 2$ and $\sigma = 2/5$ are also special, since eq. (2.6) then implies that the supersymmetric solution has zero Romans mass and, in particular, can be lifted to M-theory. Moreover, these are the endpoints of the interval where supersymmetric solutions exist (since outside this interval we would find from eq. (2.6) that $m^2 < 0$). They are indicated as vertical dashed lines in the plots.
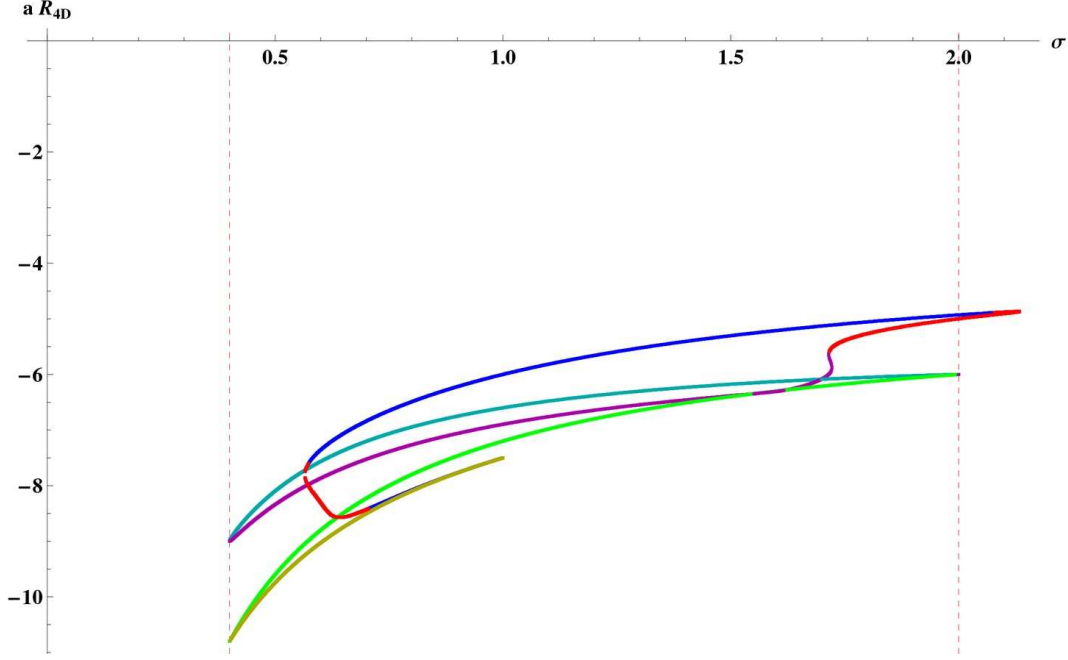
Figure 1: $\frac{\mathrm{Sp(2)}}{\mathrm{S(U(2)\times U(1))}}$-model: plot of aR_{4D} for the supersymmetric solutions (light green) and the new non-supersymmetric solutions (other colors) in terms of the shape parameter σ. Unstable solutions are indicated in red.
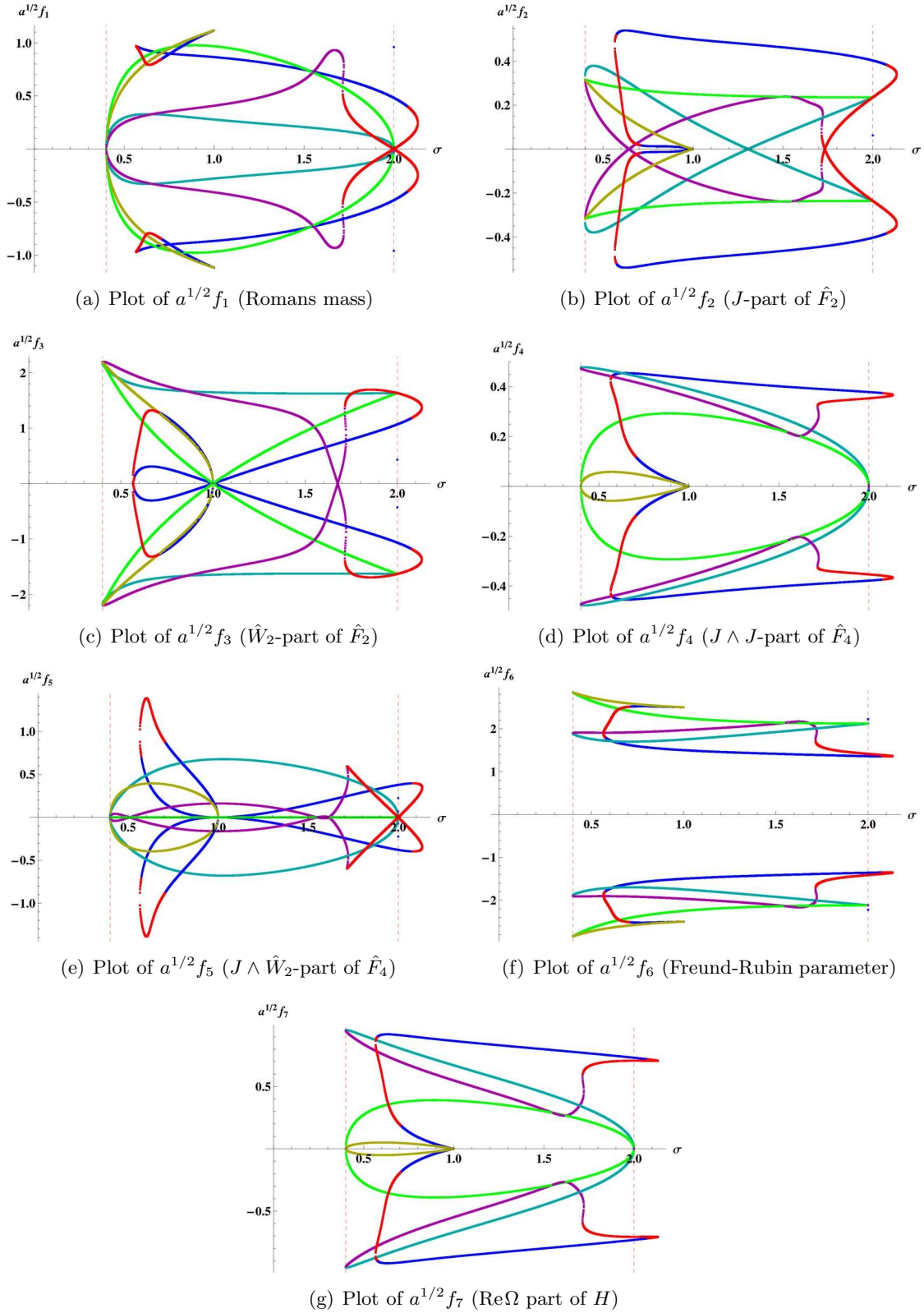
Plugging eqs. (3.5) into the supergravity equations of motion (2.9) we find numerically a rich spectrum of solutions, which are displayed in figures 1 and 2. Note that the dependence on the overall scale can be easily extracted from all plotted quantities by multiplying by a to a suitable power. We plotted the value of the 4D Ricci scalar R_{4D} of the AdS-space against the shape parameter σ in figure 1. Note that R_{4D} is inversely proportional to the AdS-radius squared and related to the effective 4D cosmological constant and the vev of the 4D scalar potential V as follows

$$\Lambda = \langle V \rangle = R_{\mathrm{4D}}/4 \,. \tag{3.6}$$

The supersymmetric solutions are plotted in light green, while red is used for the non-supersymmetric solutions found to be unstable in section 4. For completeness of the presentation of our numeric results, we provide the values of each of the coefficients f_i of the ansatz (2.2) in figure 2.

The first point to note is that where the supersymmetric solutions are restricted to the interval $\sigma \in [2/5, 2]$, there exist non-supersymmetric solutions in the somewhat larger interval $\sigma \in [0.39958, 2.13327]$. Furthermore, there are up to five non-supersymmetric solutions for each supersymmetric solution.

We remark that the parameters σ and the overall scale are not continuous moduli since they are determined by the vevs of the fluxes, which in a proper string theory treatment should be quantized. Indeed, in the next section we will show that generically all moduli are stabilized. We leave the analysis of flux quantization, which is complicated by the fact that

(a) Plot of $a^{1/2}f_1$ (Romans mass)

(b) Plot of $a^{1/2}f_2$ (J-part of $\hat{F}_2$)

(c) Plot of $a^{1/2}f_3$ ($\hat{W}_2$-part of $\hat{F}_2$)

(d) Plot of $a^{1/2}f_4$ ($J \wedge J$-part of $\hat{F}_4$)

(e) Plot of $a^{1/2}f_5$ ($J \wedge \hat{W}_2$-part of $\hat{F}_4$)

(f) Plot of $a^{1/2}f_6$ (Freund-Rubin parameter)

(g) Plot of $a^{1/2}f_7$ ($\mathrm{Re}\,\Omega$ part of H)

Figure 2: Plots of the solutions on the coset $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$. Different colors indicate different solutions. Unstable solutions are indicated in red (see section 4) and the supersymmetric solutions in light green. By a suitable rescaling of the coefficients the dependence on the overall scale a is taken out.

there is non-trivial H-flux (twisting the RR-charges), to further work. The expectation is that the continuous line of supergravity solutions is replaced by discrete solutions.

Let us now take a look at some special values of σ. For $\sigma = 1$ we find five solutions of which three (including the supersymmetric one) are up to scaling equivalent to the solutions (3.3) on $\frac{\text{G}_2}{\text{SU}(3)}$ of the previous section [35, 29, 36, 30]. They have $f_3 = f_5 = 0$ and so the fluxes are completely expressed in terms of J. However, there are also two new non-supersymmetric solutions (the dark green and the purple one) which have $f_3 \neq 0, f_5 \neq 0$.

Next we turn to the case $\sigma = 2$. This point is special in that the metric becomes the Fubini-Study metric on $\mathbb{CP}^3$ and the bosonic symmetry of the geometry enhances from $\text{Sp}(2)$ to $\text{SU}(4)$. In fact, since the RR-forms of the supersymmetric solution can be expanded in terms of the closed Kähler form $\tilde{J} = (1/3)J + (2a)^{1/2}W_2$ of the Fubini-Study metric, the symmetry group of the whole supersymmetric solution is $\text{SU}(4)$. One can also show that the supersymmetry enhances from the generic $N = 1$ to $N = 6$ [6]. In [37] it was found that there is an infinite continuous family of non-supersymmetric solutions and two discrete separate solutions (see also [35] for an incomplete early discussion), which all have $\text{SU}(4)$-symmetry. They are *not* displayed in the plot since they can not be found by taking a continuous limit $\sigma \to 2$. For these solutions $H = 0$ ($f_7 = 0$) and $\hat{F}_2$ and $\hat{F}_4$ are expanded in terms of $\tilde{J}$ (for more details see [37]).

Instead, in the plot we find apart from the supersymmetric solution (which merges with the dark green solution at $\sigma = 2$) two more discrete non-supersymmetric solutions, which have only $\text{Sp}(2)$-symmetry (since the fluxes cannot be expressed in terms of $\tilde{J}$ only). The blue one is new, while the red one turns out to be the reduction of the Englert-type solution. Indeed for the Englert-type solution we expect

$$f_1 = 0 \,, \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{no Romans mass}\,, \quad (3.7\text{a})$$

$$f_2 = f_{2,\text{susy}} \,, \quad f_3 = f_{3,\text{susy}} \,, \qquad\qquad \text{same geometry in M} \Rightarrow \text{same } \hat{F}_2 \text{ as susy}\,, \quad (3.7\text{b})$$

$$f_7 = -2f_4 = -(1/3)f_{6,\text{susy}} \,, \quad f_5 = 0 \,, \qquad\qquad\qquad \text{from } \hat{F}_4 \text{ in M-theory}\,, \quad (3.7\text{c})$$

$$f_6 = (-2/3)f_{6,\text{susy}} \,, \qquad\qquad\qquad \text{Freund-Rubin parameter changes}\,, \quad (3.7\text{d})$$

$$R_{4\text{D}} = (5/6)R_{4\text{D,susy}} \,, \qquad\qquad\qquad\qquad\qquad \text{4D } \Lambda \text{ changes}\,, \quad (3.7\text{e})$$

which agrees with the values displayed in the figures for the red curve at $\sigma = 2$.

Also for $\sigma = 2/5$ we find apart from the supersymmetric solution, the Englert solution (the purple curve) and one extra non-supersymmetric solution (the dark green curve). Note that while the supersymmetric curve joins the olive green curve at $\sigma = 2/5$, the purple curve only joins the dark green curve at $\sigma = 0.39958$.

$\frac{\text{SU}(3)}{\text{U}(1) \times \text{U}(1)}$

For this manifold the SU(3)-structure is given by [13, 14]:

$$\begin{aligned}
J &= a(-e^{12} + \rho e^{34} - \sigma e^{56}) \,, \\
\Omega &= a^{3/2}(\rho\sigma)^{1/2} \left[(e^{245} + e^{135} + e^{146} - e^{236}) + i(e^{235} + e^{136} + e^{246} - e^{145}) \right] \,,
\end{aligned} \qquad (3.8)$$

where ρ and σ are the shape parameters of the model. Furthermore we find for the torsion classes:

$$
\begin{aligned}
W_1 &= -(a\rho\sigma)^{-1/2}\frac{1+\rho+\sigma}{3}\,,\\
W_2 &= -(2/3)a^{1/2}(\rho\sigma)^{-1/2}\left[(2-\rho-\sigma)e^{12}+\rho(1-2\rho+\sigma)e^{34}-\sigma(1+\rho-2\sigma)e^{56}\right].
\end{aligned}
\tag{3.9}
$$

It turns out that the ansatz (2.7) is only satisfied for

$$
\rho=1\,,\quad \sigma=1\quad\text{or}\quad\rho=\sigma\,.
\tag{3.10}
$$

In all three of these cases the equations (2.9) for $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ reduce to exactly the same equations as for $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ so that we obtain the same solution space. However, as we will see in the next section, the stability analysis will be different since the model on $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ has two extra left-invariant modes.

In order to find further non-supersymmetric solutions, we should go beyond the ansatz (2.7). Let us put

$$
\hat{W}_2\wedge\hat{W}_2=(-1/6)\,J\wedge J+p_1\,\hat{W}_2\wedge J+p_2\hat{P}\wedge J\,,
\tag{3.11}
$$

where $\hat{P}$ is a primitive normalized (1,1)-form (so that it is orthogonal to J and $\hat{P}^2=1$). Furthermore, we also choose it orthogonal to $\hat{W}_2$ i.e.

$$
\hat{W}_2\cdot\hat{P}=0\quad\text{or equivalently}\quad J\wedge\hat{W}_2\wedge\hat{P}=0\,.
\tag{3.12}
$$

From the last equation one finds, using (2.1c), that $\mathrm{d}\hat{P}\wedge\mathrm{Im}\,\Omega=0$, which implies on $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ that

$$
\mathrm{d}\hat{P}=0\,.
\tag{3.13}
$$

One can now allow the RR-fluxes $\hat{F}_2$ and $\hat{F}_4$ to have pieces proportional to $\hat{P}$ and $\hat{P}\wedge J$ respectively and adapt the equations (2.9) accordingly to accommodate for the new contributions. Now it is possible to numerically find non-supersymmetric solutions for ρ and σ not satisfying (3.10). In particular, there are Englert-type solutions on the ellipse of values for (ρ,σ) where the supersymmetric solution has zero Romans mass. From eq. (2.6) we find that this ellipse is described by

$$
m^2=\frac{5}{16\rho\sigma}\left[-5(\rho^2+\sigma^2)+6(\rho+\sigma+\rho\sigma)-5\right]=0\,.
\tag{3.14}
$$

We will not go into more detail on these solutions in this paper.

4. Stability analysis

In this section we investigate whether the new non-supersymmetric solutions on $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ are stable[5]. To this end we calculate the spectrum of scalar fluctuations. We

---

[5]In [36] it was found that the non-supersymmetric solutions on $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ and the similar solutions on the nearly-Kähler limits of the other two coset manifolds under study are stable. We find exactly the same spectrum as the authors of that paper, which provides a consistency check on our approach. We thank Davide Cassani for providing us with these numbers, which are not explicitly given in their paper. We did not investigate the spectrum of the similar solution on $\mathrm{SU}(2)\times\mathrm{SU}(2)$, which is more complicated as there are more modes.

use the well-known result of [39, 40] that in an AdS_4 vacuum a tachyonic mode does not yet signal an instability. Only a mode with a mass-squared below the Breitenlohner-Freedman bound,

$$M^2 < -\frac{3|\Lambda|}{4}\,, \tag{4.1}$$

where $\Lambda < 0$ is the 4D effective cosmological constant, leads to an instability. We restrict ourselves to left-invariant fluctuations, which implies that even if we do not find any modes below the Breitenlohner-Freedman bound, the vacuum might still be unstable, since there might be fluctuations with sufficiently negative mass-squared that are not left-invariant. This analysis can however pinpoint many unstable vacua and we do believe it gives a valuable first indication for the stability of the others.

Truncating to the left-invariant modes on the coset manifolds under study leads to a 4D $N = 2$ gauged supergravity[6]. It has been shown in [36] that this truncation is consistent. The spectrum of the scalar fields can then be obtained from the 4D scalar potential. In fact, this computation is analogous to the one performed in [14] for the supersymmetric $N = 1$ vacua on the coset spaces. As opposed to the models here, the models in that paper included orientifolds, which broke the supersymmetry of the 4D effective theory from $N = 2$ to $N = 1$. However, also in the present case the $N = 1$ approach is applicable and effectively we have used exactly the same procedure, i.e. using the $N = 1$ scalar fluctuations and obtaining the scalar potential from the $N = 1$ superpotential and Kähler potential (see [55, 56, 57, 58]).[7] The reason is the following. The $N = 2$ scalar fluctuations in the vector multiplets are
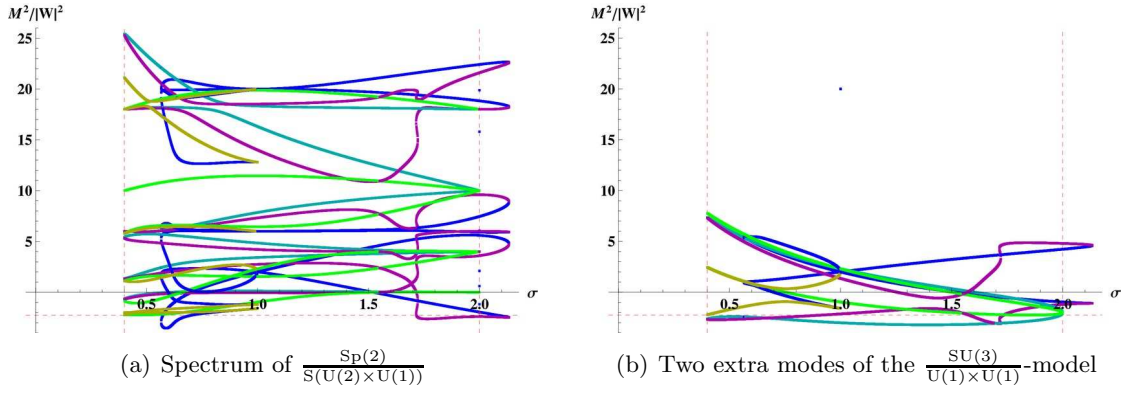
$$J_c = J - iB = (k^i - ib^i)\omega_i = t^i\omega_i\,, \tag{4.2}$$

where ω_i span the left-invariant two-forms of the coset manifold. The orientifold projection of the $N = 1$ theory would then project out the scalar fluctuations coming from expanding on *even* two-forms, which are absent for the $N = 1$ theory on the coset manifolds under study. The scalar fluctuations in the $N = 2$ vector multiplets are thus exactly the same as the scalars in the chiral multiplets of the Kähler moduli sector of the $N = 1$ theory. The

---

[6]It is important to make the distinction between the number of supersymmetries of respectively the 4D effective theory, the 10D compactifications, and their 4D truncation (which are the solutions of the 4D effective theory [36]). In the presence of one left-invariant internal spinor, the effective theory will be $N = 2$ since this same spinor can be used in the $4 + 6$ decomposition of both ten-dimensional Majorana-Weyl supersymmetry generators, but multiplied with independent four-dimensional spinors. On the other hand, for a certain compactification to preserve the supersymmetry, certain differential conditions, which follow from putting the variations of the fermions to zero must be satisfied. In the presence of RR-fluxes, these conditions mix both ten-dimensional Majorana-Weyl spinors, putting the four-dimensional spinors in both decompositions equal. A generic supersymmetric compactification therefore only preserves $N = 1$. The $\sigma = 2$ supersymmetric Kähler-Einstein solution on $\mathbb{CP}^3$ on the other hand is non-generic in that it preserves $N = 6$, of which only one internal spinor is left-invariant under the action of $\mathrm{Sp}(2)$ and remains after truncation to 4D.

[7]It is interesting to note that (in $N = 1$ language) all the D-terms vanish, so that the supersymmetry breaking is purely due to F-terms. Indeed, in [58] it is shown that $\mathcal{D} = 0$ is equivalent to $\mathrm{d}_H(e^{2A-\Phi}\mathrm{Re}\,\Psi_1) = 0$ in the generalized geometry formalism. For SU(3)-structure this translates to $\mathrm{d}(e^{2A-\Phi}\mathrm{Re}\,\Omega) = 0$ and $H \wedge \mathrm{Re}\,\Omega = 0$, which is satisfied for our ansatz, eq. (2.1) and (2.2).

(a) Spectrum of $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ (b) Two extra modes of the $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$-model

Figure 3: Spectrum of left-invariant modes of the solutions on $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$.

expansion forms can then be chosen to be the same as the $Y_i^{(2-)}$ of [14]. Furthermore, there is one tensor multiplet, which contains the dilaton Φ, the two-form $B_{\mu\nu}$ and two axions ξ and $\tilde{\xi}$ coming from the expansion of the RR-potential C_3:

$$C_3 = \xi\,\alpha + \tilde{\xi}\,\beta\,, \tag{4.3}$$

where a choice for α and β spanning the left-invariant three-forms would be $Y^{(3-)}$ and $Y^{(3+)}$ of [14] respectively. In the presence of Romans mass or $\hat{F}_2$-flux the two-form $B_{\mu\nu}$ becomes massive and cannot be dualized to a scalar. The dilaton and $\tilde{\xi}$ appear in a chiral multiplet of the complex moduli sector of the $N = 1$ theory, while $B_{\mu\nu}$ and ξ are projected out by the orientifold. By using the $N = 1$ approach we thus loose the information on just one scalar ξ. A proper $N = 2$ analysis would however learn that ξ does not appear in the scalar potential (see e.g. [36]), implying that it is massless and thus above the Breitenlohner-Freedman bound. Moreover, the scalar potential should be the same whether it is obtained directly from reducing the 10D supergravity action (as in [59]) or whether it is obtained using $N = 2$ or $N = 1$ technology[8]. Furthermore we note that the massless scalar field ξ not appearing in the potential is not a modulus, since it is charged [60, 61], and therefore eaten by a vector field becoming massive.

The spectra of left-invariant modes for $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ are displayed in figure 3. The Breitenlohner-Freedman bound is indicated as a horizontal dashed line. The Sp(2)-model has six scalar fluctuations entering the potential: k^i, b^i with $i = 1, 2$ from the two vector multiplets, and $\Phi, \tilde{\xi}$ from the universal hypermultiplet, while the SU(3)-model has two more fluctuations from the extra vector multiplet. These two extra modes make a big difference for the stability analysis since one of them tends to be below the Breitenlohner-Freedman bound for the purple and dark green solution. As a result, even though the solutions for the Sp(2)- and SU(3)-model take the same form, the SU(3)-model has more unstable solutions: compare figure 1 and 4.

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[8]The only potential difference between the latter two would be the contribution from the orientifold. We have checked that this contribution vanishes in the scalar potentials of [14] in the limit of the orientifold charge $\mu \to 0$.
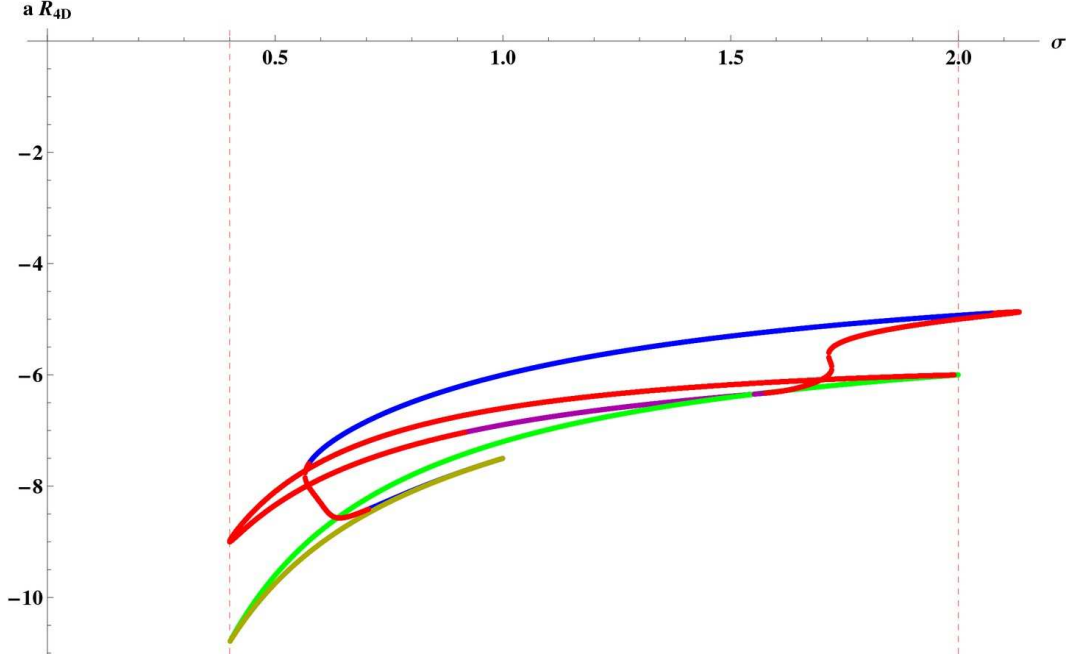
Figure 4: $\frac{\text{SU}(3)}{\text{U}(1)\times\text{U}(1)}$-model: plot of $aR_{4\text{D}}$ in terms of the shape parameter σ. Unstable solutions are indicated in red.

In particular, we note that the reduction of the Englert-type solution is unstable for $\sigma = 2$ in the Sp(2)-model, in agreement with [38], since the M-theory lift of the corresponding supersymmetric solution has eight Killing spinors. We indeed find the same negative mass-squared $M^2 = -(4/5)|\Lambda|$ for the unstable mode as in that paper. On the other hand, for $\sigma = 2/5$ the Englert-type solution is stable against left-invariant fluctuations. This is still in agreement with [38] which relied on the existence of at least two Killing spinors, while the M-theory lift of the $N = 1$ supersymmetric solution at $\sigma = 2/5$ has only one Killing-spinor. For the SU(3)-model, all Englert-type solutions turn out to be unstable (including the ones outside the condition (3.10)).

We also investigated the stability of the additional solutions at the special point $\sigma = 2$ found in [37]. We found that for the Sp(2)-model all these solutions are stable against left-invariant fluctuations. For the SU(3)-model on the other hand it turns out that the discrete solutions in eqs. (3.16) and (3.17) of that reference are unstable, while the continuous family of eq. (3.18) becomes unstable for

$$\frac{\gamma^2}{\beta^2} > \frac{5(75 \mp 16\sqrt{21})}{8217}\,, \tag{4.4}$$

for the $\pm$ sign choice in front of the square root in eq. (3.18) of that paper respectively (note that the supersymmetric solution corresponds to the point $\gamma^2/\beta^2 = 0$ in this family).

Finally, we note that generically (i.e. unless an eigenvalue is crossing zero at a special value for σ) all the plotted modes are massive. For a range of values for σ one of the eigenvalues for the dark green and purple solution takes a small, but still non-zero value.

5. Conclusions

In this paper we presented new families of non-supersymmetric AdS$_4$ vacua. In fact, extrapolating from our analysis on these specific coset manifolds and under the assumption that a proper treatment of flux quantization does not kill much more vacua than in the supersymmetric case, it would seem that there are more of these non-supersymmetric vacua than supersymmetric ones. This would imply that such vacua cannot be ignored in landscape studies. We have moreover shown that many of them are stable against a specific set of fluctuations, namely the ones that can be expanded in terms of left-invariant forms. If these vacua turn out to be stable against all fluctuations they should also have a CFT-dual, which could be studied along the lines of [20], where the three-dimensional Chern-Simons-matter theory dual to a particular highly symmetric non-supersymmetric vacuum was proposed. Furthermore, the nice property of some IIA vacua that all moduli enter the superpotential and thus can be stabilized at a classical level [15] also extends to our non-supersymmetric vacua.

A next step would be to relax the constraint that the solutions should have the same geometry as the supersymmetric solution. It is also interesting to investigate whether a similar ansatz and techniques can be used to look for tree-level dS-vacua [62].

Acknowledgments

A. SU(3)-structure

A real non-degenerate two-form J and a complex decomposable three-form Ω define an SU(3)-structure on the 6D manifold M_6 iff:

$$\Omega \wedge J = 0 \,, \tag{A.1a}$$

$$\Omega \wedge \bar{\Omega} = \frac{8i}{3!} J \wedge J \wedge J \neq 0 \,, \tag{A.1b}$$

and the associated metric is positive-definite. This metric is determined by J and Ω as follows:

$$g_{mn} = -J_{mp} I^p{}_n \,, \tag{A.2}$$

with I the complex structure associated (in the way of [63]) to Ω. The volume-form is given by $\mathrm{vol}_6 = \frac{1}{3!} J^3 = -(i/8)\Omega \wedge \bar{\Omega}$.

The intrinsic torsion of the manifold M_6 decomposes into five torsion classes $\mathcal{W}_1, \ldots, \mathcal{W}_5$. Alternatively they correspond to the SU(3)-decomposition of the exterior derivatives of J and Ω [64]. Intuitively, they parameterize the failure of the manifold to be of special holonomy, which can also be thought of as the deviation from closure of J and Ω. More specifically we have:

$$\begin{aligned}
\mathrm{d}J &= \frac{3}{2}\mathrm{Im}\,(\mathcal{W}_1\bar{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \, , \\
\mathrm{d}\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \bar{\mathcal{W}}_5 \wedge \Omega \, ,
\end{aligned}$$

(A.3)

where $\mathcal{W}_1$ is a scalar, $\mathcal{W}_2$ is a primitive (1,1)-form, $\mathcal{W}_3$ is a real primitive $(1,2)+(2,1)$-form, $\mathcal{W}_4$ is a real one-form and $\mathcal{W}_5$ a complex (1,0)-form. In this paper only the torsion classes $\mathcal{W}_1$, $\mathcal{W}_2$ are non-vanishing and they are purely imaginary, so it will be convenient to define $W_{1,2}$ so that $\mathcal{W}_{1,2} = iW_{1,2}$. A primitive (1,1)-form P (such as W_2) transforms under the **8** of SU(3) and satisfies

$$P \wedge J \wedge J = 0 \, .$$

(A.4)

The Hodge dual is given by

$$\star_6 P = -P \wedge J \, .$$

(A.5)

A primitive $(1,2)$(or $(2,1)$)-form Q on the other hand transforms as a **6** (or $\bar{\mathbf{6}}$) under SU(3) and satisfies

$$Q \wedge J = 0 \, .$$

(A.6)

B. Type II supergravity

The bosonic content of type II supergravity consists of a metric G, a dilaton Φ, an NSNS three-form H and RR-fields F_n. We use the democratic formalism of [65], in which the number of RR-fields is doubled, so that n runs over $0, 2, 4, 6, 8, 10$ in type IIA and over $1, 3, 5, 7, 9$ in IIB. We will often collectively denote the RR-fields with the polyform $F = \sum_n F_n$. We have also doubled the RR-potentials, collectively denoted by $C = \sum_n C_{(n-1)}$. These potentials satisfy $F = \mathrm{d}_H C + me^{-B} = (\mathrm{d} + H\wedge)C + me^{-B}$. In type IIB there is of course no Romans mass m, so that the second term vanishes. In type IIA we find in particular $F_0 = m$.

The bosonic part of the pseudo-action of the democratic formalism then simply reads

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}X\sqrt{-G}\left\{e^{-2\Phi}\left[R + 4(\mathrm{d}\Phi)^2 - \frac{1}{2}H^2\right] - \frac{1}{4}F^2\right\} \, ,$$

(B.1)

where we defined $F^2 = \sum_n F_n^2$ and the square of an l-form P as follows

$$P^2 = P \cdot P = \frac{1}{l!}P_{m_1\ldots m_l}P^{m_1\ldots m_l} \, ,$$

(B.2a)

where the indices are raised with the inverse of the metric G_{mn} or the internal metric g_{mn} (defined later on), depending on the context. In the following it will also be convenient to define:

$$P_m \cdot P_n = \iota_m P \cdot \iota_n P = \frac{1}{(l-1)!}P_{mm_2\ldots m_l}P_n{}^{m_2\ldots m_l} \, .$$

(B.2b)

The extra degrees of freedom for the RR-fields in the democratic formalism have to be removed by hand by imposing the following duality condition at the level of the equations of motion after deriving them from the action (B.1):

$$F_n = (-1)^{\frac{(n-1)(n-2)}{2}} \star_{10} F_{10-n} \,. \tag{B.3}$$

That is why (B.1) is only a pseudo-action.

The fermionic content consists of a doublet of gravitinos ψ_M and a doublet of dilatinos λ. The components of the doublets are of different chirality in type IIA and of the same chirality in type IIB.

In this paper we look for vacuum solutions that take the form $\mathrm{AdS}_4 \times M_6$. In principle there could also be a warp factor A, but it will always be constant for the solutions in this paper. We can choose it to be zero. The compactification ansatz for the metric then reads

$$\mathrm{d}s_{10}^2 = G_{mn}\mathrm{d}X^m\mathrm{d}X^n = \mathrm{d}s_4^2 + g_{mn}\mathrm{d}x^m\mathrm{d}x^n \,, \tag{B.4}$$

where $\mathrm{d}s_4^2$ is the line-element for AdS_4 and g_{mn} is the metric on the internal space M_6. For the RR-fluxes the ansatz becomes

$$F = \hat{F} + \mathrm{vol}_4 \wedge \tilde{F} \,, \tag{B.5}$$

where $\hat{F}$ and $\tilde{F}$ only have internal indices. The duality constraint (B.3) implies that $\tilde{F}$ is not independent of $\hat{F}$, and given by

$$\tilde{F}_n = (-1)^{\frac{(n-1)(n-2)}{2}} \star_6 \hat{F}_{6-n} \,. \tag{B.6}$$

What we need in this paper are the type II equations of motion, which can be found from the pseudo-action (B.1). We use them as they are written down in [5] (originally they were obtained for massive type IIA in [35]), but take some linear combinations in order to further simplify then. Without source terms (i.e. we put $j_{\mathrm{total}} = 0$ in the equations of motion of [5]), they then read:

$$\mathrm{d}_H F = 0 \qquad\qquad\qquad \text{(Bianchi RR fields)}\,, \tag{B.7a}$$

$$\mathrm{d}_{-H} \star_{10} F = 0 \qquad\qquad \text{(eom RR fields)}\,, \tag{B.7b}$$

$$\mathrm{d}H = 0 \qquad\qquad\qquad \text{(Bianchi H)}\,, \tag{B.7c}$$

$$\mathrm{d}\left(e^{-2\Phi} \star_{10} H\right) - \frac{1}{2}\sum_n \star_{10} F_n \wedge F_{n-2} = 0 \qquad \text{(eom H)}\,, \tag{B.7d}$$

$$2R - H^2 + 8\left(\nabla^2\Phi - (\partial\Phi)^2\right) = 0 \qquad \text{(dilaton eom)}\,, \tag{B.7e}$$

$$2(\partial\Phi)^2 - \nabla^2\Phi - \frac{1}{2}H^2 - \frac{e^{2\Phi}}{8}\sum_n nF_n^2 = 0 \qquad \text{(trace Einstein/dilaton eom)}\,, \tag{B.7f}$$

$$R_{MN} + 2\nabla_M\partial_N\Phi - \frac{1}{2}H_M \cdot H_N - \frac{e^{2\Phi}}{4}\sum_n F_{nM}\cdot F_{nN} = 0 \tag{B.7g}$$

$$\text{(Einstein eq./dilaton/trace)}\,.$$

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